Noise Characteristics of Combustion Augmented Jets at Midsubsonic Speeds

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Noise generation by a subsonic flow discharging from a combustion chamber is examined with regard to the relative importance of combustion as a source of noise in such a flow system. Measurements of pressure fluctuations inside the combustor are compared with far-field noise measurements by direct cross-correlations. The cross-correlations and derived cross-spectral densities verify that much of the noise originates inside the combustor. A fluid mechanic perturbation model is used to predict exit plane velocity fluctuations due to internal pressure fluctuations. This unsteadiness at the exit plane is assumed to behave as an acoustic monopole which radiates to the far field. Far-field noise levels estimated on this basis are in good agreement with measured values. The over-all noise level from the combustor/jet is found to be 10–20 db higher than for an equivalent clean, cold jet at the same exit velocity, in the range of 500–700 fps.

Introduction

IT is well known that unsteady combustion can amplify and generate pressure oscillations in the gaseous medium surrounding the zone of combustion and that these oscillations can be in the audible frequency range. Noise from furnaces and other turbulent flames give evidence to this fact. No general theory or scaling law exists, however, which will allow reliable predictions to be made concerning the importance of combustion as a source of noise in a flow regime such as is found in an aircraft jet engine. Some initial results of a study to provide such information are contained in this paper.

Noise generation by open flames has been the subject of several studies during the past twenty years. Giammar and Putnam[1] have presented a review of the experimental work in this area. Several interesting features of combustion noise may be derived from these earlier studies. The characteristic frequencies of combustion noise are found to be considerably lower than for jet noise. Putnam[2] showed, for example, that the noise from his fuel jet with no combustion peaked at around 10 kHz whereas, with the jet burning, the spectrum between 100 and 500 Hz was dominant and at a level of about 20–30 db higher than the peak of the nonburning case. The point of the spectral sound pressure level peak was found to depend upon the type of flame[2] as well as the type of fuel and diameter of the burner,[3,4] and the ratio of burner diameter to flow velocity.[5] This Strouhal type scaling suggests that the noise from open turbulent flames is related to the turbulence scale in the flame; the dependence on fuel type suggests some link between the noise generation mechanisms and the chemistry of the reaction. Any theory on combustion noise should therefore contain both of these parameters.

Theoretical work concerning combustion noise has received some attention, but again, mostly related to open flames. Bragg[6] developed a theory on the basis of physical reasoning using the wrinkled laminar flame concept, a concept which is open to question.[7] Kotake and Hatta[5] have also developed a theory for combustion noise. They interpreted observed sound power scaling with flow velocity, U, to the fourth power, to be due to dipole radiation, an interpretation which does not agree with derivations made by Lighthill.[8] Strahle[9] provided a critique of existing theories of combustion noise and developed two alternative theories of his own. His theories follow more directly from Lighthill's[8] formulation than do the others, and he uses the two existing mechanistic theories for turbulent flames, the wrinkled laminar flame theory and the distributed reaction theory, to derive acoustic efficiencies for flames. In each case he utilizes a proportionality constant which should be near unity in value if the model is accurate. In comparing his theories with the experiments of Smith and Kilham,[3] he finds the proportionality constants to be 124 and 0.17, respectively, for the wrinkled flame and distributed reaction models. It is evident therefore, that further sophistication or more accurate representations of the noise producing mechanisms in flames is clearly needed before accurate predictions of their potential as a noise source can be made.

It is not surprising, then, to find that the importance of combustion as a source of noise in jet engines is not well understood. The previous experiments on open flames bear little resemblance to the ducted burner system in an engine, and theories which are inadequate to describe open flames are certainly inadequate to predict the noise from internal combustion. Further, if we examine noise data from a wide range of aircraft engines,[10] it is not at all clear that combustion, or some other internal source, is not an important source of noise in engines, at least at jet speeds below 1000 fps, since the over-all noise scaling with velocity does not follow the classical U^8 law in this range of velocity, as clean laboratory jets may be observed to do. Therefore, further study of the relative importance of combustion as a source of noise in jet exhausts is needed. The results presented in this paper are some initial results obtained in a program of research aimed at providing answers to the questions raised in this paragraph.

Present Investigations

The aim of this investigation was to demonstrate the relative importance of combustion as a source of noise in a flow regime representative of a subsonic jet engine exhaust. This paper describes the results of measurements of unsteadiness inside a 3-in.-diam combustor which exhausts through a 2-in.-diam

Presented as Paper 73-189 at the AIAA 11th Aerospace Sciences Meeting, Washington, D.C., January 10–12, 1973; submitted March 8, 1973; revision received October 4, 1973. This paper is based on work performed under NASA Grant NGR 31-001-241, issued by the Acoustics Branch of NASA Langley Research Center, Hampton, Va.

Index categories: Aircraft Noise, Powerplant; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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Table 1 Combustor operating conditions

Combustor-condition	A/F ratio	Jet Mach number	Exhaust gas total temp. (°R)	Combustion efficiency (%)	Jet vel. (fps)	Internal over-all RMS roughness (db)
A-1	40	0.288	2163	96.9	654	158
A-2	50	0.290	1810	90.9	603	159
A-3	40	0.220	2078	90.28	528	157
A-4	50	0.243	1742	85.18	496	158
B-1	40	0.288	2124	95.76	649	158
B-2	50	0.289	1793	89.5	598	159
B-3	40	0.238	2092	90.05	531	157
B-4	50	0.247	1650	77.89	470	158
C-1	40	Unstable				
C-2	50	0.3037	1980	100.0[a]	660	156
C-3	40	0.2407	2199	97.2	551	160
C-4	50	0.258	1956	100.0[a]	557	155

[a] The temperature was calculated from gas dynamic relations using measured mass flow and pressure drop through the nozzle, assuming a 100% efficient nozzle. The temperature computed on this basis may be expected to be higher than the actual temperature, hence resulting in a computed combustion efficiency which is too high.

nozzle, over a velocity range of 500–700 fps, and the results of measurements of the far-field noise due to the combustor/jet combination as well as correlation of the data. Within the scope of this paper, the origin of combustion noise will not be discussed further, but will be accepted as given by measurements obtained inside the combustor. These measurements will be examined with respect to their basic characteristics, such as frequency content and intensity as compared with the far-field noise measurements which contain the contribution made by the jet in addition to the noise from the combustor. Analytic expressions are derived and used to predict the far-field noise due to the unsteadiness in the combustor. Results of predicted noise levels are compared with measured values.

Details of Experiment

In view of the fact that a relatively quick, quantitative demonstration of the relative importance of combustion noise was sought, without an immediate understanding of the detailed mechanisms involved, an experiment was designed for this purpose. A 3-in. i.d. combustor was fitted with a 2-in. i.d. nozzle and connected by flexible tubing to metered air and fuel supplies. The combustor assembly was mounted on a pylon 12 ft above the ground level to minimize ground reflections of the noise. Six microphones stationed at 15° intervals between 15° and 90° to the jet axis, also mounted 12 ft above the ground, at a distance of 50 ft from the combustor exit were used to measure the far-field noise.

Figure 1 shows the details of the combustors used. In combustor "A" the fuel is injected into a tube which is mounted concentrically within the main combustor. Near the downstream end of the can is a V-gutter type flame holder which generates turbulence to stabilize the flame. Liquid iso-octane is the fuel used in all the experiments reported in this paper. A mixture of hydrogen and air is injected and spark ignited near the flame holder to start the iso-octane combustion. Pressure transducer ports are located at four stations along the combustor. Only a single transducer, located immediately upstream of the converging nozzle, was used for measurements reported in this paper.

Combustor "B" consisted of combustor "A" with an 8-in. extender inserted between the flame holder and the nozzle, as shown in Fig. 1. Combustor "C" has a can which is slightly larger in diameter and longer than the tube on "A," with perforations along the side to allow the air to enter. No primary air enters the upstream end of the can, and the flame is stabilized by a recirculating flow induced at the upstream end of the can. Table 1 summarizes the operating conditions used.

During a given experiment, air and fuel flow rates and the nozzle pressure ratio were measured, all to be used for tem-

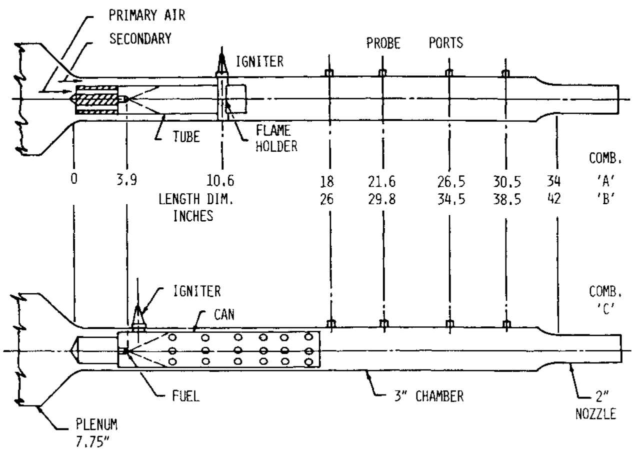

Fig. 1 Schematic of combustors used showing important dimensions. Combustors "A" and "B" are identical except that "B" is 8 in. longer than "A"; combustor "C" has no primary air, but secondary air enters combustor can through side wall perforations.

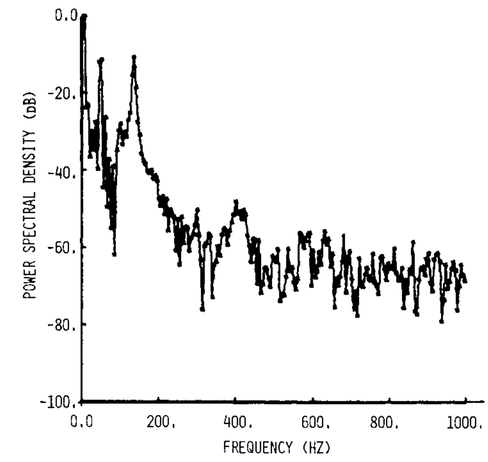

Fig. 2 Power spectral density distribution of chamber pressure, combustor "A," $A/F = 40$, $M_j \simeq 0.29$.

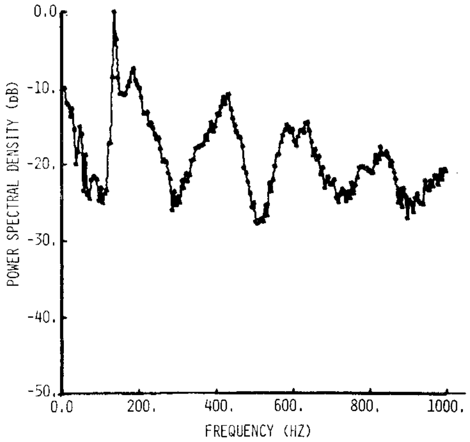

Fig. 3 Power spectral density distribution of far-field noise, at 50 ft, 60° from jet axis, combustor "A," $A/F = 40$, $M_j \simeq 0.29$.

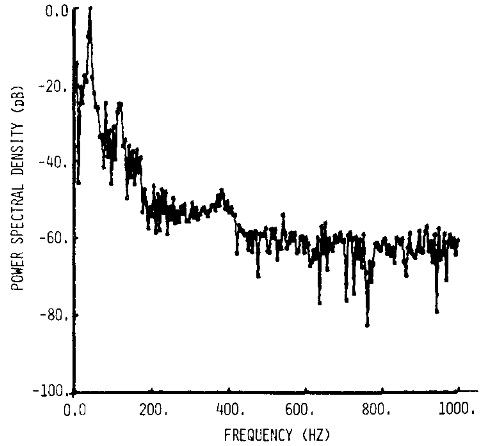

Fig. 5 Power spectral density of combustion chamber pressure, combustor "A," $A/F = 50$, $M_j \simeq 0.24$.

perature and efficiency calculations. Noise measurements consisted of simultaneously recording the signal from the pressure transducer (Kistler type 601L1) in the combustor along with the signal from five B&K type 4135, $\frac{1}{4}$-in. microphones, on magnetic tape, using a Honeywell 7600 tape recorder. For signal analysis, the tape was replayed at a later time.

Data extracted from the records included sound pressure levels (using a B&K Model 2305 Sound Level Recorder), probability distribution functions, and auto-correlations of the individual signals, as well as cross-correlations between the pressure fluctuations measured inside the combustor and the far-field noise measurement, all three using a SAICOR Model 43-A, 400 point Correlation and Probability Analyzer. The auto and cross-correlations were digitized using a DATACOM Model 8015 A to D converter and power spectral and crosspower spectral density distributions were obtained using a Fast Fourier Transform routine on an IBM 360/91 Computer. Some sample data will now be discussed.

Discussion of Results

Spectral Content of Noise

Figure 2 shows a plot of the power spectral density distribution of the chamber pressure obtained for operating condition 1 (see Table 1), combustor "A." This plot is the result of taking the Fourier transform of the autocorrelation of the pressure transducer signal. Note the spectrum peaks at 5, 50, and 135 Hz, with a sharp drop in the power spectral density above 135 Hz. There is some evidence of a peak at around 400 Hz as well as around 600 Hz. Figure 3 is the corresponding power spectral density distribution of the microphone signal, for the microphone at 50 ft from the nozzle exit and at 60° from the jet axis. The maximum in this spectrum corresponds to the 135 Hz peak seen inside the combustor. Other peaks are seen at 5, 185, and 420 Hz, with some double peaks around 600 and 800 Hz. If the peaks observed in the microphone spectrum are due to the oscillations inside the combustor at the corresponding frequencies, then it appears as though the ultra-low frequencies are transmitted through the nozzle less efficiently than the higher frequencies. This is what may be expected on the basis of acoustic theory.[11] To determine further if the peaks in the microphone spectrum above 300 Hz might originate inside the combustor, a 300 Hz high pass filter was used on the chamber pressure record to cut out the intense low frequency oscillations and thus allow amplification of the higher frequencies during playback of the recorded signal, without over driving the amplifiers. Figure 4 is the resulting power spectral density distribution of the chamber pressure. The frequencies of the peaks correspond closely with those measured by the microphone, although the microphone signal has a more coalesced form of several double peaks compared with the chamber pressure spectrum. This may be expected due to the coalescing nature of successive pressure pulses. This changing wave shape may also account for the slight shift in frequencies of some of the higher frequency peaks.

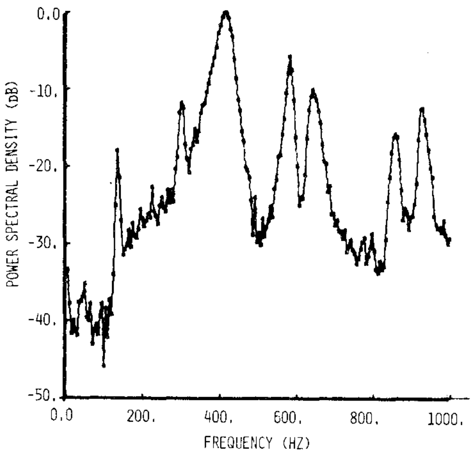

Fig. 4 Power spectral density distribution of chamber pressure with 300 Hz high pass filter. Combustor "A," $A/F = 40$, $M_j \simeq 0.29$.

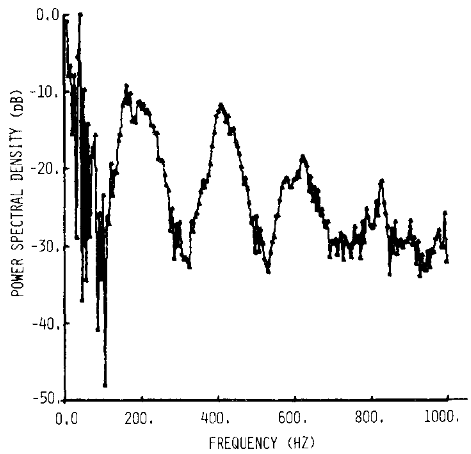

Fig. 6 Power spectral density of far-field noise (60°, 50 ft), combustor "A," $A/F = 50$, $M_j \simeq 0.24$.

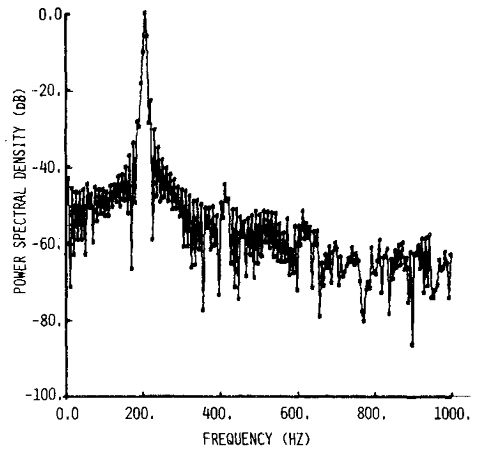

Fig. 7 Power spectral density of combustor pressure, combustor "C," $A/F = 40$, $M_j = 0.24$.

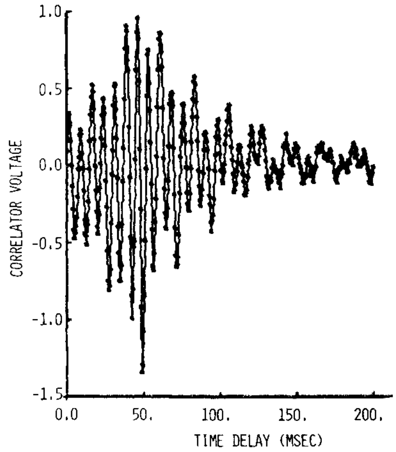

Fig. 9 Cross-correlation of chamber pressure and far-field noise (60°, 50 ft), combustor "A," $A/F = 40$, $M_j \simeq 0.29$.

Figures 5 and 6 show the power spectral density distributions of the chamber pressure and far-field microphones respectively for combustor "A," condition 4. The peak in both spectra appear at 40 Hz for this case, with the harmonic-like oscillations again appearing in the microphone signal. A change in the air/fuel ratio and the flow rate has resulted in a change in the dominant frequencies present in the spectrum.

Figure 7 shows the power spectral density of the chamber pressure obtained for combustor "C," condition 3. The 205 Hz spectral peak is very dominant in this spectrum. Figure 8 is the power spectral density distribution for the microphone signal for this same case, again showing the dominant 205 Hz signal with some weaker harmonic-like peaks at 410, 615, and 820 Hz. The modification in the burner geometry has resulted in a different power spectral density distribution.

A direct verification of the link between the internal chamber pressure fluctuations and the far-field noise was obtained by real time cross-correlations of the signals. Figure 9 shows the cross-correlation for condition 1, combustor "A." The peak in the cross-correlation at about 45 msec time delay corresponds to the time for the acoustic disturbance to travel from inside the combustor to the far-field microphone, 50 ft away. The cross-spectral density distribution, Fig. 10, shows the same dominant frequencies seen in Figs. 3 and 4 for the individual pickups, but with less resolution in the harmonics.

From examination of these spectral records, it is evident that noise from a combustion system is most dominant at low frequencies. The low frequencies observed here are believed to be related to duct resonance modes which have been excited by the unsteady combustion. The 205 Hz peak in combustor "C" corresponds approximately to the $\frac{1}{4}$ wave tube excitation in the cavity downstream of the flame front. The 135 Hz peak with combustor "A" similarly corresponds approximately to its $\frac{1}{4}$ wave tube excitation. The very low frequency (40 Hz) observed in combustor "A," condition 4, Fig. 5, is clearly too low for a $\frac{1}{4}$ wave tube excitation, and its origin has not been located to date, but since it appears in both the chamber pressure signal, Fig. 5, and in the microphone signal, it is definitely an acoustic type oscillation.

Quantitative Examination of Results

The directivity pattern of the noise generated by the three combustors operating at $A/F = 40$ and $M_j = 0.24$ is shown in Fig. 11. At this condition, combustor "C" is considerably noisier than combustors "A" or "B," the latter two being essentially equally noisy. The dominant noise produced by combustor "C" and radiated to the far field was at approximately 205 Hz, at every azimuth angle. To separate the contribution of the combustion process to the far-field noise from that of other noise sources, combustor "C" was operated with ambient temperature air flow at a jet exit Mach number of 0.25. Comparison between the directivity pattern of the noise produced by the cold air flow and those of combustors "A," "B," and "C"

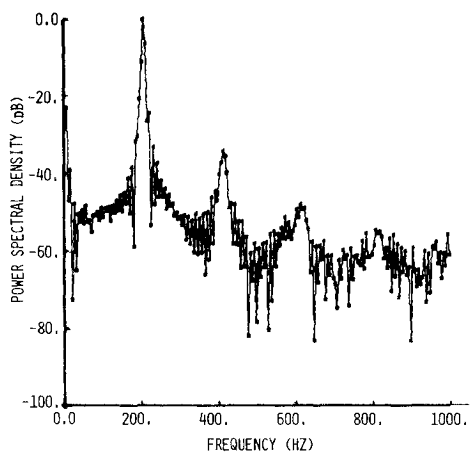

Fig. 8 Power spectral density of far-field noise (60°, 50 ft), combustor "C," $A/F = 40$, $M_j \simeq 0.24$.

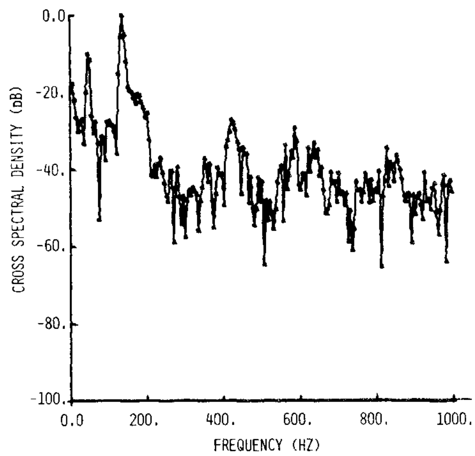

Fig. 10 Cross-spectral density of chamber pressure and far-field noise correlation (60°, 50 ft), combustor "A," $A/F = 40$, $M_j = 0.29$.

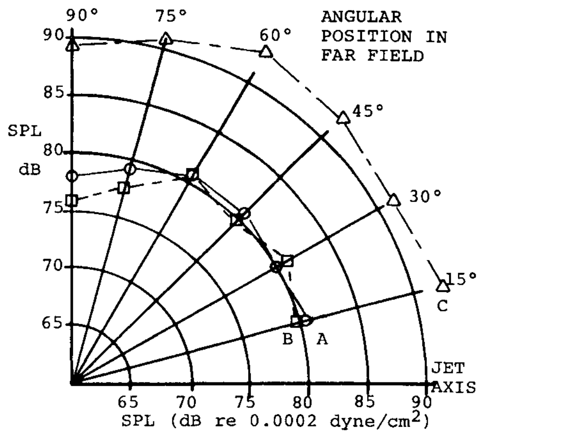

Fig. 11 Sound pressure level in far field for three combustors at approximately same operating conditions; $A/F = 40$, $M_j \simeq 0.24$.

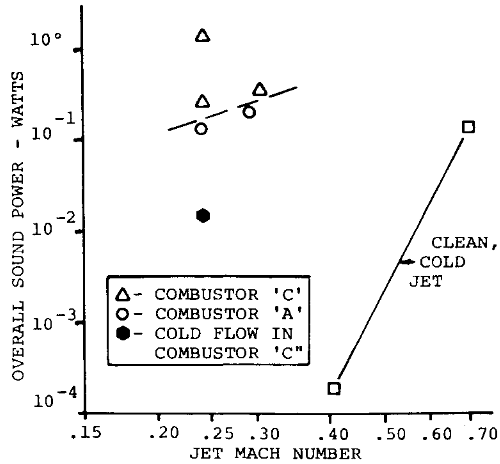

Fig. 13 Comparison of over-all sound power of combustor-jet with that of equivalent clean jet with 2-in.-diam nozzle, using Mach number (equal thrust) as base of comparison.

operated at $A/F = 50$ and the same exit Mach number of 0.25 is shown in Fig. 12. The difference in noise level for the case with combustion compared with the case with no combustion, at the same Mach number, is about 14 db. The noise from this cold flow is undoubtedly higher than for a clean jet, due to the flow obstructions inside the duct but still 14 db lower than the equivalent case with combustion. The dominant frequency for this cold flow was in the vicinity of 4000 Hz while at the same Mach number, in combustor "C," the hot flow had a dominant frequency of 205 Hz.

Figure 13 shows the over-all sound power obtained from measurements on combustors "A" and "C" compared with the sound power from a clean jet of corresponding size, plotted with Mach number on the abscissa. Comparison on the basis of equal Mach number implies equal thrust. On this basis, if it was possible, a clean cold jet could generate substantially more thrust with much less noise. Or, stating the comparison another way, for a given jet Mach number, the flow originating in a combustion chamber generates several orders of magnitude more noise than a flow exhausting from a smooth duct with no combustion. The hexagonal point in the center of Fig. 13 reminds one that the obstructions, such as the burner can and struts inside the combustor, account for some of the excess noise even in the absence of combustion. Nevertheless, the case with combustion is still 14 to 20 db noisier than the cold flow case in the same combustor, at the same Mach number.

Figure 14 shows the same comparison on the basis of jet velocity. On this basis there is less difference between the clean cold jet and the jet originating inside a combustion chamber. The noise from the combustors is now only 10 to 20 db in excess of the jet noise.

These two figures (Figs. 13 and 14) very emphatically demonstrate that combustion upstream of a jet nozzle will increase the noise power (for the same jet velocity) by a factor of between 10 and 100, depending on the roughness level inside the combustor. An analytical relationship to link the roughness inside the combustor to the noise level produced outside is needed. Some attempts have been made to produce such a relationship. The following section outlines the approach taken to date.

Analytical Predictions of Noise Due to Combustor Roughness

It would be of considerable interest simply to be able to monitor the pressure fluctuations inside a combustion chamber and by some analytical procedure predict reliably what the noise level outside would be. We may find after some further examination that we need to know both the temperature and pressure fluctuations in the combustor, but for a beginning we will try using only the pressure.

Two steps are envisioned in this prediction scheme. Since the

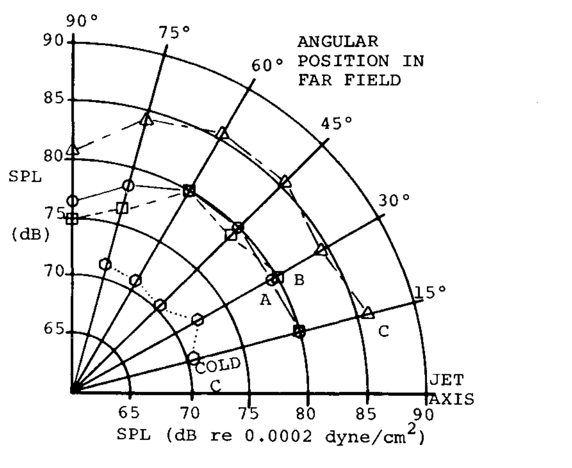

Fig. 12 Sound pressure level in far field at approximately same operating conditions with three combustors. A/F ratio $= 50$, $M_j \simeq 0.25$.

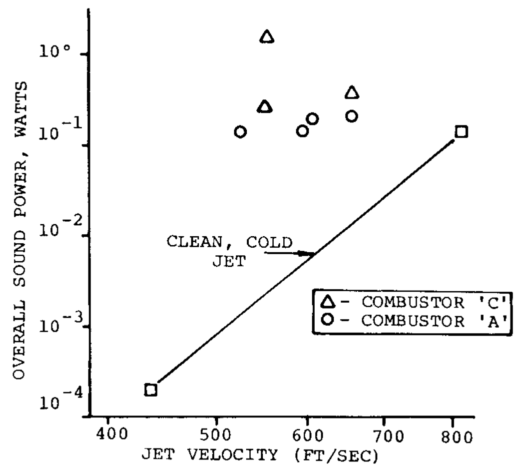

Fig. 14 Comparison of over-all sound power of combustor-jet with that of equivalent clean jet with 2-in.-diam nozzle, using jet velocity as base of comparison.

sound generated by internal sources is best evaluated in terms of a surface integral of the appropriate quantities across the exit plane of the exhaust nozzle, the first step will be to predict what these terms will be at the exit plane, using measurements inside the combustor, and the second step will be to calculate the noise generated at the exit plane. From Curle's[12] general solution, we know that the over-all noise generated by a jet is given by

$$p' = \frac{1}{4\pi c^2}\frac{x_i x_j}{|\bar{x}_i|^3}\frac{\partial^2}{\partial t^2}\int_{V_0}[T_{ij}]\,dV_0 - \frac{1}{4\pi c}\frac{x_i}{|\bar{x}_i|^2}\frac{\partial}{\partial t}\oint_{S_0}[\rho v_i v_j + p_{ij}]\cdot dS_0 + \frac{1}{4\pi}\frac{1}{|\bar{x}|}\frac{\partial}{\partial t}\oint_{S_0}[\rho v]\cdot dS_0 \tag{1}$$

Here the first term represents the jet noise sources and the other two terms are integrals over all adjacent bounding surfaces which might contribute to the noise. The jet exit plane is clearly one such surface. The third term in Eq. (1), is believed in the present case, to be the dominant one. Therefore, attempts have been made to evaluate the noise due to mass flow fluctuations at the exit plane caused by unsteadiness inside the combustor.

Assuming a constant density, inviscid flow between the chamber and nozzle exit, Bernoulli's equation, applicable along streamlines, may be applied in the form

$$\rho(\partial u/\partial t) + \rho\Delta\tfrac{1}{2}u^2 = -\nabla p \tag{2}$$

For this quasi-steady, perturbation solution, the integral along the streamlines will neglect the first term in Eq. (2) resulting in Eq. (3)

$$\tfrac{1}{2}\rho u_2{}^2 + p_2 = \tfrac{1}{2}\rho u_1{}^2 + p_1 \tag{3}$$

where subscript 1 will denote conditions in the combustor and 2 will denote exit plane conditions. Then assuming a perturbation type solution with $u_1 = u_{01}$, $u_2 = u_{02} + u_2'$, $p_2 = p_{02}$ and $p_1 = p_{01} + p_1'$, Eq. (3) may be reduced to a quasi-steady and a perturbed solution. The perturbed form may be written as

$$u_2' = p_1'/\rho_0 u_{02} \tag{4}$$

Using Eq. (4), the exit plane velocity fluctuations may now be evaluated from chamber pressure fluctuations. If the exit plane is now considered to be a simple monopole, the noise in the far field can be shown to be[13]

$$p_f = [\rho_0 u_2' \omega a^2/r(2)^{1/2}] \tag{5}$$

where $\omega = 2\pi f$ is the angular frequency of the fluctuation, a is the nozzle radius and r is the distance to the far-field point.

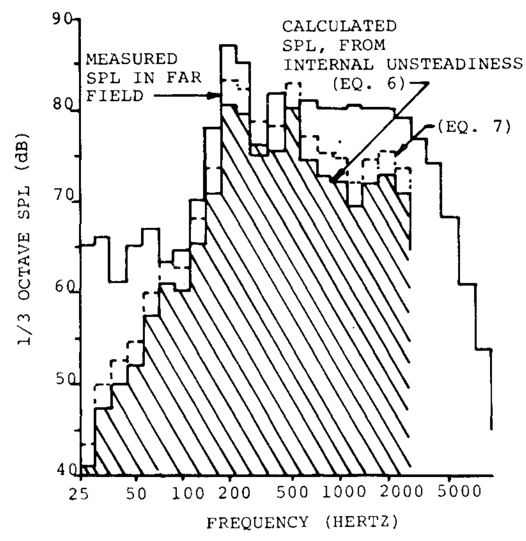

Fig. 15 $\frac{1}{3}$ octave analysis of measured SPL in far field, compared with calculated values; combustor "C," Condition 2.

Using results of a $\frac{1}{3}$ octave analysis of the pressure fluctuations inside combustor "C" for operating condition 2 as the input to calculations, the far-field noise in each band is calculated after combining Eqs. (4) and (5)

$$p_f = [p_1'/(2)^{1/2}](\omega a/u_{02})\cdot(a/r) \tag{6}$$

Figure 15 shows the comparison of the measured far-field sound and the calculated levels using the aforementioned theory. The shaded curve is the one calculated in this way. The agreement in the shape of the curve is quite good but the magnitude of the calculated noise level is about 6 db lower than the measured value. One further observation should be made. Equation (5) is derived for a source in a constant temperature region all the way to the measuring point. If a temperature change occurs between these points, the acoustic energy transmitted will be equal to the incident energy times a transmission coefficient, where the transmission coefficient is evaluated based on the impedance change.[11] For this case, assuming ideal gas behavior, the pressure fluctuation will be enhanced by the temperature drop. The modified expression is given by Eq. (7)

$$p_f = \frac{p_1'}{(2)^{1/2}}\left(\frac{\omega a}{u_{02}}\right)\left(\frac{a}{r}\right)\left(\frac{2(T_2/T_f)^{1/2}}{(T_2/T_f)^{1/2}+1}\right) \tag{7}$$

This additional factor accounts for an increase of about 2.4 db in the calculated levels, as shown in Fig. 15 with the dotted curve. Now the agreement is better than before with some deviation at low and high frequencies. The low frequency deviation is probably due to the outdoor background noise which is about 60 to 65 db. The high frequency deviation may be due to jet noise. One would expect the peak of the jet noise to occur in the frequency range around 1000 to 3000 Hz, which is the range of this excess noise. From Fig. 14 one would not expect the jet noise to be anywhere near the combustor noise, but from Fig. 15 it appears to be within close enough range to be noticed. This could indicate that the level of jet noise can be increased by a high level of unsteadiness upstream of the nozzle. If this is the case, engine noise which has been attributed to jet noise in the past may have been influenced by internal sources through an increase in the intensity of the jet noise.

Summary and Conclusions

In summary, the following observations have been made:

1) The power spectral density content of combustion generated noise has been observed to be dominantly in the low frequency range, with several spectral peaks below 1000 Hz, corresponding to the $\frac{1}{4}$ wave tube cavity excitation and several harmonics.

2) The same narrow band spectral peaks appearing inside the combustion chamber appear in the far-field noise signal. This is verified in cross-spectral densities of the two signals.

3) Comparison of noise levels from the combustor exhaust with clean jet noise levels, either on a basis of jet velocity or jet Mach number, show the combustor exhaust to be much noisier.

4) The noise level of the combustor exhaust depends upon the fuel/air ratio, the combustor geometry and the over-all mass flow through the combustor.

5) Attempts to analytically predict the far-field noise from the internal pressure fluctuations, in a simple manner, were quite successful. A more sophisticated theory is, however, needed to permit predictions to be made over a wide range of geometries and operating conditions.

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MARCH 1974 AIAA JOURNAL VOL. 12, NO. 3

Hypersonic Merged Stagnation Shock Layers Part I: Adiabatic Wall Case

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The study of the merged layer flow in the stagnation region of a blunt body with continuum approach is the subject of the present investigation. Emphasis is laid on understanding the effects of slip boundary conditions on the flowfield. Full Navier-Stokes equations are solved using the concept of local similarity for a wide range of Reynolds numbers and Mach numbers. Detailed comparison of our results with other theoretical and experimental investigations is carried out. It is found that at sufficiently low Reynolds numbers, existing theories with thin-layer approximations fail to give correct results. Agreement of our theoretical predictions of impact pressure, density profile, and recovery temperature with the available experimental data is reasonably good.

Nomenclature

A	$= C_p T_{o\infty}/V_\infty{}^2$
C_H	= heat-transfer coefficient
C_p, C_v, γ	= specific heat at constant pressure, volume, and the ratio C_p/C_v
K	= thermal conductivity
Kn	= Knudsen number
M_∞	= freestream Mach number
p	= pressure
p_2	= auxiliary pressure, Eq. (1)
p_i	= impact pressure
p_o'	= ideal inviscid impact pressure
Pr	= Prandtl number
r, r_B	= radius, body radius
Re	$= \rho_\infty V_\infty r_B/\mu_{o\infty}$
Re_∞	$= \rho_\infty V_\infty r_B/\mu_\infty$
Re_S	$= \rho_\infty V_\infty r_B/\mu_S$
T	= temperature
$T_{o\infty}$	= freestream adiabatic stagnation temperature
u, v	= tangential and radial velocity components
V_∞	= freestream velocity
α, σ	= thermal accommodation and reflection coefficients
μ	= viscosity
ρ	= density
τ	= skin friction
λ	= mean free path

Subscripts

AW	= pertaining to adiabatic wall temperature
FM	= pertaining to the free molecular theory
w	= pertaining to the body surface conditions
∞	= pertaining to freestream conditions
S	= conditions behind shock wave

Received April 20, 1973; revision received September 13, 1973. This work was supported by the Ministry of Defence, India. One of us (ACJ) wishes to thank W. K. Dahm, Chief, Aerophysics Division, MSFC, for his support and kind encouragement to write this manuscript.

Index categories: Supersonic and Hypersonic Flow; Rarefied Flows; Viscous Nonboundary-Layer Flows.

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